Multiple Roll Systems: Steady-State Operation

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Roll coaters for applying liquid coating to continuous strip or web are, with some two-roll exceptions, systems of three or more rolls in which liquid passes through two or more gaps or nips between rolls. Yet most of the literature on roll coating is devoted to some of the 11 distinct flows in individual gaps or nips. This article analyzes how the final coated layer thickness in several types of forward roll transfer and reverse roll coating systems depends, at steady state, on the number of rolls, their speeds, the gaps between roll pairs, and the doctoring of recycle films from the rolls. The inputs to the analysis are elementary mass balances at the gaps, and simple gap performance equations that approximate well the available experimental and theoretical findings about flow rates and film-splits at individual gaps. The results are fundamentals-based means of understanding, comparing, predicting, and ultimately designing performance of multiple roll systems.

Introduction

Roll coaters perform the four basic functions of coating: to feed, meter, distribute, and apply a liquid film onto a continuous substrate, by using a system of counter- and/or corotating rolls. Common configurations of roll coaters have from two to six rolls—historically as many as thirty or more—to accomplish these coating functions. It is important to know, both from an operational standpoint and from a design standpoint, how multiple roll coaters behave: how they create thin films, how they respond to changes in process variables (such as roll speeds, roll gaps, and liquid properties) and how they should be controlled to meet the target coat weight (or liquid volume per unit of substrate area). To understand and predict the behavior of multiple roll coaters requires analysis on two levels. First are the flows in *individual* gaps or nips between pairs of rolls, that is, the fundamental flows of roll coating. Second are the *systems* of gap and nip flows that are the essence of coaters with more than two rolls. In this article, *gap* refers to the clearance between rolls that are not in interference at rest, while *nip* refers to the liquid-filled clearance between those that are. In places, the term *gap* is used for both.

There are eleven fundamental roll coating flows at individual gaps; these are shown in Figure 1. Some of the most important have been the subjects of extensive theoretical and experimental studies. These studies have produced correlations of flow rates, flow rate limits, and film-split ratios in single gaps and nips. In this article, we investigate how individual gaps combine and interact with each other in roll systems, that is, combinations of three or more rolls, and hence two or more gaps. We compile available experimental and theoretical findings about flow rates and film-splits at individual gaps, and represent them by simple gap performance equations. Gap performance equations and mass balance equations together constitute models of multiple roll systems. We link the individual gaps together by mass balances: an outlet film of one gap serves as an inlet film to the next gap.

We examine several types of forward roll transfer coaters and reverse roll coaters as examples to show how the final coated film thickness depends on operating parameters in steady-state operation. In a sequel article, we examine dynamic responses to step changes in process conditions and to continuous perturbation by roll runout.

Forward Roll Transfer Coaters

Forward roll coaters utilize a series of counterrotating rolls and successive film-splits to meter and apply a film of liquid onto a continuous substrate. The two-roll, forward coater is the simplest and one of the earliest forms (Tranquir, 1926). It uses just one forward film-splitting nip to meter and apply the liquid coating. A large and varied assortment of much

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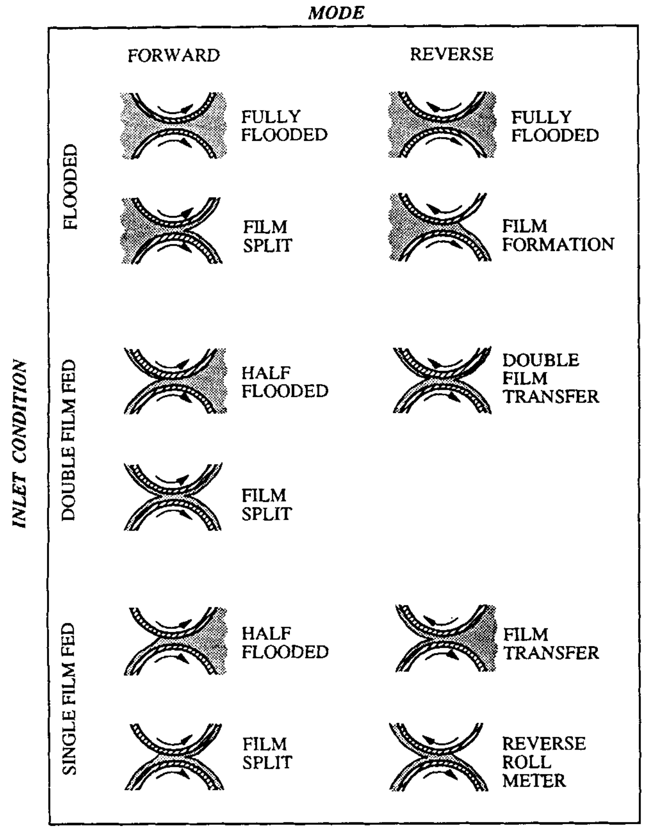


Figure 1. Eleven distinct flows between a pair of rolls: the fundamental flows of roll coating.

more complex transfer coaters, once popular in the 1930s through the 1950s in the coated-paper industry, utilized from fifteen to thirty or more forward roll nips to coat concentrated suspensions of clay (Massey et al., 1938; Kaulakis, 1974). These seem extinct today, but three-, four- and five-roll descendants of transfer coaters survive and are prevalent in coil coating, that is, precoating continuous strips of metal, and in coating ultrathin (≤ 1 micron wet) silicone release materials on paper and polymer films. Another popular type of multiple, forward roll coater is the offset gravure configuration that uses one or more intermediate rolls between the gravure roll and the backing roll to transfer the coating liquid. All of these systems have in common a series of multiple forward roll gaps and nips.

Some of the reasons cited for using a series of rolls are to "impart shear" to the liquid; to "smooth the coating" prior to application; to provide a means of speedup between the slower moving gate rolls and the faster moving substrate, that is, the line speed; and to produce a thin liquid by multiple film-splits. But there is little published information on how these types of coaters work: how a thin film is created; how sensitive the coat weight is to changes in roll speeds, roll gaps, and liquid properties; and how best to control the coat weight. Brea and Williamson (1992) addressed one type of transfer coater with the goal of estimating the shear rates in the individual gaps; they did not address the issue of how rapidly the effects of shearing strain relax as the liquid is carried from one gap to the next by a roll, however.

Multiple roll systems are also employed in the ink distribution systems of printing presses. In these systems as many as ten to fifteen rolls are commonly used, and the rolls are often arranged in branched configurations, as opposed to the linear configurations of multiple roll transfer coaters. Mill (1961) examined the steady-state and transient behavior of a fifteen roll ink distribution system both theoretically and experimentally. He considered the special case in which all rolls turned at the same speed, and he limited most of his analysis to symmetric film-splits. Later, Hull (1968) made similar steady-state calculations of multiple roll ink distributions systems with the goal of minimizing abrupt variations of ink density or "ghosting" on the printing roll. MacPhee (1979) made calculations of the ink film thicknesses on all of the intermediate rolls in a ten-roll ink distribution system. The aforementioned studies of ink distribution systems consider branched configurations of rolls in which the rolls turn at the same speed and the film-splits are symmetric. Here, we consider only linear arrangements of rolls, but include the cases of unequal roll speeds and unsymmetric film-splits.

Theory

Ordinary transfer coaters

Two types of equations describe a series of forward roll film-splits: mass balances, one per gap, and film-split ratio equations, one per forward roll film-split. For the *ordinary transfer roll system* of n rolls and $n-1$ gaps depicted in Figure 2, where all of the liquid films are recycled between gaps,

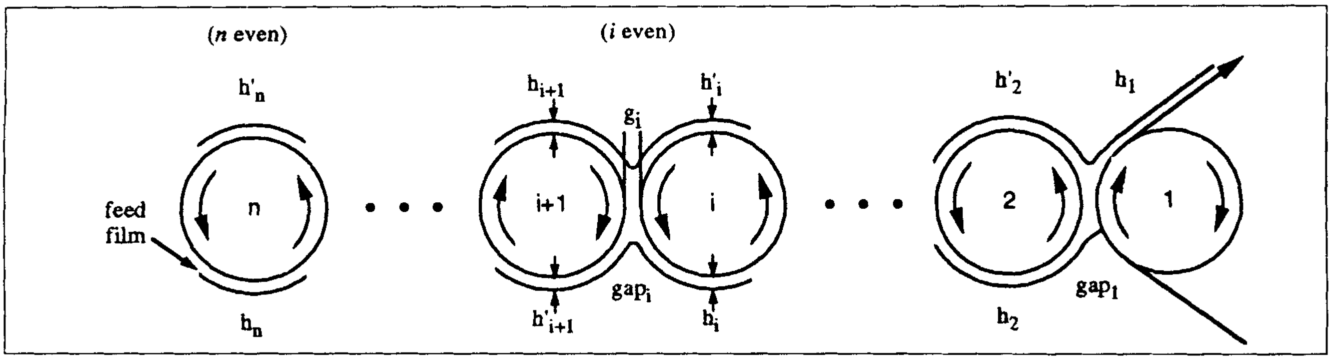


Figure 2. A series of n counterrotating rolls, forming n-1 gaps.

The recycled films, h_i', are not doctored. Odd rather than even values of n and i change the direction of the roll rotation only.

the mass balance at the application gap (gap_1 between rolls 1 and 2) is

$$V_2 h_2 = V_2 h_2' + V_1 h_1,$$
$$(in) = (out) \quad (1)$$

where V_i is the surface speed of roll i, and h_i is the film flow rate per unit width Q_i divided by V_i; $h_i \equiv Q_i/V_i$. Provided the gaps are far enough apart so that film flow is fully developed and the effects of gravity and roll curvature are negligible, as they ordinarily are, the film is in plug flow and h_i is equal to the film thickness. The mass balance can be simplified and rewritten in terms of roll speed *ratios*:

$$V_{2,1}(h_2 - h_2') - h_1 = 0, \quad (2)$$

where $V_{i,j}$ represents the ratio of the speed of roll i to the speed of roll j, that is, $V_{i,j} = V_i/V_j$. The formulas here and throughout the article pertain to situations where no coating liquid overflows or is otherwise lost from the ends of the gaps or rolls.

All of the intermediate rolls (roll 2 through roll n) have two films on them, one that travels toward the substrate, h_i, and one that travels away from the substrate, h_i'. The latter is a *recycle* film. At the intermediate gaps (gap_2 through gap_{n-1}) the mass balance is different because there are two incoming films and two outgoing films:

$$V_{i+1,i}(h_{i+1} - h_{i+1}') - (h_i - h_i') = 0 \qquad 2 \le i \le n-1. \quad (3)$$

The same mass balance follows from balancing the inputs and outputs of an individual roll and noting that the net flow rate on every roll, Q_{net}, is equal to the rate at which the liquid is being coated, namely $V_1 h_1$. Thus

$$V_1 h_1 = Q_{net} = V_i(h_i - h_i') \qquad 2 \le i \le n. \quad (4)$$

Setting equal the net flow rates on any two adjacent rolls, say roll i and roll $i+1$, leads to the mass balance of Eq. 3.

The film-split ratio in a forward roll film-split between two smooth and impermeable rolls is approximated well by a power-law relationship:

$$\frac{h_{i+1}'}{h_i} = \alpha_i \left(\frac{V_{i+1}}{V_i} \right)^{\beta_i} = \alpha_i (V_{i+1,i})^{\beta_i} \qquad 1 \le i \le n-1. \quad (5)$$

The values of α_i and β_i depend strongly on the direction and strength of gravity in the gap relative to the viscous and inertial forces (Coyle et al., 1986; Benjamin, 1994). Depending on the liquid properties, the roll diameter, and the gap width, β_i varies between 0.3 and 0.7. When the rolls are aligned horizontally, α_i is equal to unity, and when the rolls are aligned vertically, α_i varies between 0.65 and 1.5. When the liquid velocity is shear sensitive (non-Newtonian), α_i and β_i become functions of the shear rate. If the shear-rate dependence is described by a power law, the film-split parameters are functions of the power law index as well (Coyle et al., 1987; Benkreira et al., 1981). At this writing, there is limited information, theoretical or experimental, about the film-split behavior between a rigid and a deformable roll (or two deformable rolls), and between a roll and a rough, permeable substrate like paper, which may also be appreciably deformable.

Equations 2, 3, and 5 provide $2(n-1)$ equations: $n-1$ mass balances, and $n-1$ film-split equations, that must be solved for $2(n-1)$ unknowns: h_i ($i = 1, n-1$), and h_i' ($i = 2, n$). This set of equations can be solved algebraically, and the final film thickness and all intermediate film thicknesses can be written as functions of the inlet film thickness h_n and the parameters $V_{i,1}$ ($i = 2, n$), α_i, and β_i ($i = 1, n-1$). Perhaps the most useful result is the equation for h_1, for it shows how that thickness depends on the operating parameters. From the formulas for h_1 when $n = 2, 3, 4$, and 5, a general equation for h_1 as a function of n follows by induction:

$$h_1 = \frac{h_n V_{n,1}}{1 + \sum_{i=1}^{n-1} \prod_{j=n-i}^{n-1} \alpha_j (V_{j+1,j})^{1+\beta_j}}. \quad (6)$$

Expressions for the intermediate film thicknesses can be derived readily by starting with the formula for h_1 and successively substituting it into the film-split equation (Eq. 5) and the mass balance equations (Eqs. 2 and 3).

A basic assumption in the mass balance equations is that enough liquid is delivered to the gap so that it does not starve and break the coating bead, yet not so much liquid is delivered to the gap that it floods and rejects liquid. For two liquid films delivered to the gap to merge and pass through, the gap widths, g_i ($i = 1, n-1$) must be set to satisfy the following condition:

$$\frac{1}{\lambda_{i_{max}}} \le \frac{g_i(1 + V_{i+1,i})}{2(V_{i+1,i} h_{i+1} + h_i')} \le \frac{1}{\lambda_{i_{min}}}, \quad (7)$$

where $\lambda_{i_{min}}$ and $\lambda_{i_{max}}$ are the minimum and maximum flow rates that can pass through a double-film-fed, forward-roll gap, as determined by experiment and/or theory (cf. Benjamin, 1994).

Doctored transfer coaters

Another version of transfer coater is that depicted in Figure 3. In this system, all of the recycle films, h_i' ($i = 2, n$) are scraped or doctored; the liquid so removed may be returned to the feed system. Removing this liquid from the rolls prevents it from becoming trapped indefinitely in a recirculating flow around the roll as it does in the ordinary transfer roll system. For such a *doctored transfer roll system*, the mass balance at intermediate gaps is different because only one film enters:

$$V_{i+1,i}(h_{i+1} - h_{i+1}') - h_i = 0 \qquad 1 \le i \le n-1. \quad (8)$$

The mass balance at the application gap (gap_1) is still given by Eq. 2. But the expression for the coated film thickness, h_1, differs slightly from Eq. 6; it is

$$h_1 = \frac{h_n V_{n,1}}{\prod_{i=1}^{n-1} \left[1 + \alpha_i (V_{i+1,i})^{1+\beta_i} \right]}. \quad (9)$$

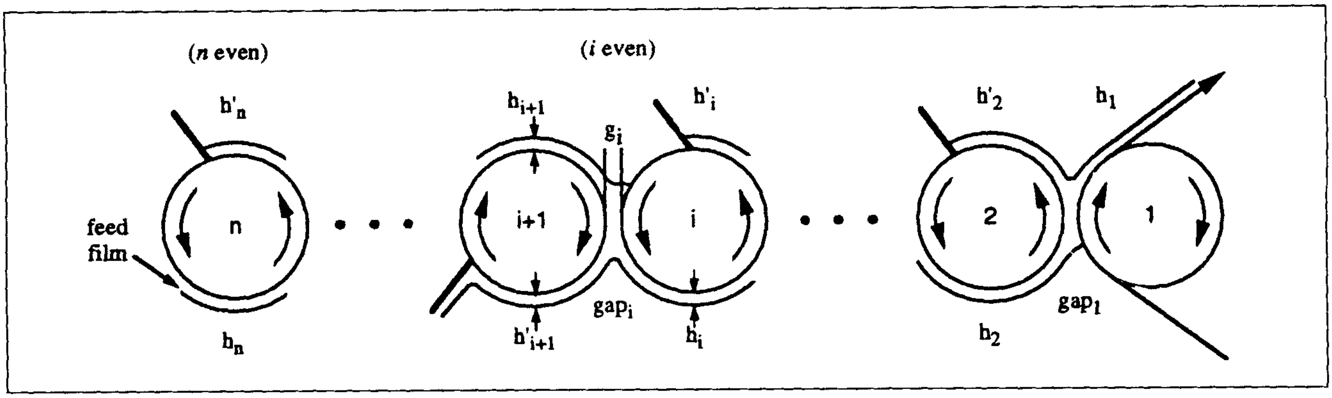


Figure 3. A series of n counterrotating rolls, where all of the recycle films, h_i', are removed from the rolls by a scraper or doctor blade.
Odd rather than even values of n and i change the direction of the roll rotation only.

The criterion for the gap settings, g_i, differs slightly from Eq. 7; it is

$$\frac{1}{\lambda_{i_{\max}}} \leq \frac{g_i(1+V_{i+1,i})}{2(V_{i+1,i}h_{i+1})} \leq \frac{1}{\lambda_{i_{\min}}}. \tag{10}$$

To complete the description of a multiple, forward-roll coater, an expression for the feed film, h_n, is needed. This depends on the type of feed system used.

Feed systems

Any one of a number of feed systems may be coupled to a forward-roll system. Some of these are shown in Figure 4. Each gives rise to a different formula for the feed film h_n.

Forward Roll Pond Feed. In a gap feed or pond feed, an added metering roll, $n+1$, forms a metering gap (gap$_n$). The last two rolls, roll n and roll $n+1$, are commonly called the gate rolls and are run slowly to avoid air entrainment, splashing, and mixing, all of which can raise froth and foam on a pond or in a pan. The liquid is fed to the pond at the same rate, on average, that it is being carried away by the coated substrate. This is known as a demand feed system and in practice is accomplished with either a level control device at the pond or an overflow from the pond.

If neither gate roll is deformable, the flow rate through the gap is proportional to the average roll speed and to the gap or minimum clearance, g_n, between the two rolls:

$$h_n + V_{n+1,n}h_{n+1} - \frac{\lambda_n g_n}{2}(1+V_{n+1,n}) = 0. \tag{11}$$

The proportionality constant λ_n has been found to be between 1.2 and 1.4 for double-film-fed forward-roll coating flows, depending on the depth of the pond (Benjamin, 1994). Combining Eq. 11 with the film-split formula, Eq. 5, yields an expression for the feed film thickness:

$$h_n = \frac{\lambda_n g_n(1+V_{n+1,n})}{2\left[1+\alpha_n(V_{n+1,n})^{1+\beta_n}\right]}. \tag{12}$$

More commonly, one of the gate rolls is covered with a deformable material like rubber or another elastomer. In this case the minimum clearance between the gate rolls is not as easily defined. The minimum clearance in the feed nip in its deformed state, g_n, is a function of the elastic moduli and thickness of the elastomeric cover, the liquid properties, the roll speeds, and the loading force between the rolls. With this

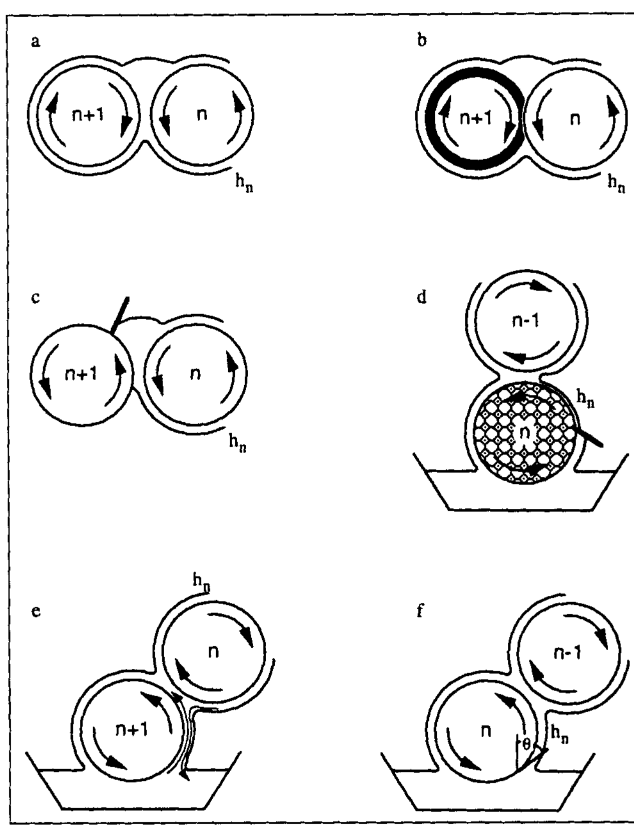


Figure 4. Several different feed systems used with a multiple roll coater.
(a) Rigid roll pond feed. (b) Deformable roll pond feed. (c) Reverse roll pond feed. (d) Gravure roll. (e) Pan feed with runback. (f) Pan feed without runback.

definition of g_n, λ_n varies between 1.1 and 1.4. Currently such relationships are available from only a few studies of deformable roll coating nips (Coyle, 1988; Carvalho and Scriven, 1993).

Reverse Roll Pond Feed. In an alternative pond feed, the end gate rolls, that is, roll n and roll $n+1$, rotate in the same angular direction, so that the feed film is created by a reverse-roll metering action. In this alternative, both gate rolls are typically rigid rolls (Kosta, 1977). An equation for the feed film thickness in the case of a half-flooded metering gap is given in the discussion of the multiple reverse-roll coaters below; it is Eq. 30.

Pan Feed. Perhaps the most common way to feed a roll coater is to rotate a roll, sometimes called the pickup roll, in a pan of the coating liquid. This roll is rotated slowly to avoid entraining air into the liquid and to avoid splashing and surging in the pan. Most often, the liquid on the pickup roll is in a state of runback, that is, more liquid is dip-coated onto the roll than can pass through the first gap, and the excess runs back into the pan. In this case, the gap is actually metering the flow, and the equation for the feed film is that given for the pond feed, Eq. 11, with λ_n between 1.3 and 1.37 (Kubota and Scriven, 1993; Benkriera et al., 1981; Schneider, 1962). The pickup roll is roll $n+1$, and the feed film, h_n, is not that created by dipping from the pan, but is the film leaving the first gap as illustrated in Figure 4e.

If the liquid on the pickup roll is not in runback and all of the liquid picked up from the pan passes through the first gap, then the metering action is like that of dip coating. The feed film is that dipped from the pan, and the thickness of this film can be estimated with a result from dip-coating theory (Landau and Levich, 1942; Deryagin and Levi, 1964):

$$h_n = 0.944\left(\frac{\mu V_n}{\sigma}\right)^{1/6}\left[\frac{\mu V_n}{\rho g(1-\sin\theta)}\right]^{1/2} \tag{13}$$

provided $\mu V_n/\sigma < 10^{-2}$. The feed film is a function of the absolute speed of roll n, V_n; gravity, g; the angle of inclination of the roll as it leaves the liquid, θ; and the physical properties of the liquid: σ is the surface tension, μ is the viscosity, ρ is the density. In this case the pickup roll is roll n because it delivers the feed film.

Gravure Roll Feed. Offset gravure coaters use a gravure roll to feed the transfer rolls. In this case, the feed film is actually the liquid contained in the etched pattern of the gravure roll. The effective feed film thickness is the volume factor (cell volume per unit surface area of the roll) of the gravure or knurl pattern. The film-split action between a gravure roll and a smooth roll is somewhat different from that between two smooth rolls, but the general equation for film-split ratio, Eq. 5, can be expected to describe it fairly well with appropriate values of α_i and β_i.

Analysis of pond-fed transfer coaters

To show how these formulas apply we examine in detail some of the more commonly used transfers coaters, namely, two-, three-, four-, and five-roll, pond-fed transfer coaters. With the film-split parameters chosen to be the same at every split ($\alpha_i = 1$, $\beta_i = \beta$), the equations for the coated film thickness simplify considerably:

Two-Roll, Pond-Fed, Undoctored

$$h_1 = \frac{\lambda_1 g_1(1+V_{2,1})}{2\left[1+(V_{2,1})^{1+\beta}\right]}, \tag{14}$$

Three-Roll, Pond-Fed, Undoctored

$$h_1 = \frac{V_{2,1}\lambda_2 g_2(1+V_{3,2})}{2\left[1+(V_{3,2})^{1+\beta}\right]\left[1+(V_{2,1})^{1+\beta}\right]}, \tag{15}$$

Four-Roll, Pond-Fed, Undoctored

$$h_1 = \frac{V_{3,1}\lambda_3 g_3(1+V_{4,3})}{2\left[1+(V_{4,3})^{1+\beta}\right]\left[1+(V_{3,2})^{1+\beta}+(V_{3,1})^{1+\beta}\right]}, \tag{16}$$

Five-Roll, Pond-Fed, Undoctored

$$h_1 = \frac{V_{4,1}\lambda_4 g_4(1+V_{5,4})}{2\left[1+(V_{5,4})^{1+\beta}\right]\left[1+(V_{4,3})^{1+\beta}+(V_{4,2})^{1+\beta}+(V_{4,1})^{1+\beta}\right]}. \tag{17}$$

The relationship $V_{i,j} = V_{i,k}/V_{j,k}$ has been used to simplify the foregoing equations.

The equation for the film thickness h_1 is the same for the doctored and undoctored transfer coaters with two and three rolls because these coaters have no recycle films in either case. Four- and five-roll coaters do have recycle films, and the formulas for the final coated film thickness reflect this. With the film-split parameters set to $\alpha_i = 1$ and $\beta_i = \beta$ the equations for doctored transfer coaters become

Four-Roll, Pond-Fed, Doctored

$$h_1 = \frac{V_{3,1}\lambda_3 g_3(1+V_{4,3})}{2\left[1+(V_{4,3})^{1+\beta}\right]\left[1+(V_{3,2})^{1+\beta}\right]\left[1+(V_{2,1})^{1+\beta}\right]}, \tag{18}$$

Five-Roll, Pond-Fed, Doctored

$$h_1 = \frac{V_{4,1}\lambda_4 g_4(1+V_{5,4})}{2\left[1+(V_{5,4})^{1+\beta}\right]\left[1+(V_{4,3})^{1+\beta}\right]\left[1+(V_{3,2})^{1+\beta}\right]\left[1+(V_{2,1})^{1+\beta}\right]}. \tag{19}$$

Table 1. Survey of Industrial Practitioners and Available Literature Reveals Large Differences in Preferred Roll Speed Settings for Forward Roll Transfer Coaters

3-Roll	Booth (1970)	Hebels (1990)	Zink (1979)	Potjer (1992)	
$V_{2,1}$	1	0.7–0.95	0.75	0.65–0.98	
$V_{3,1}$	0.067	0.15–0.25	0.1	0.25–0.4	
4-Roll	Jacobs (1963)	Satas (1984) Weiss (1977)	Zahn (1993)	Booth (1970)	Alheid (1978)
$V_{2,1}$	0.85–0.9	1	0.95–2	1	1
$V_{3,1}$	0.4	0.25	0.5–1	1	0.5–0.9
$V_{4,1}$	0.4	0.1	0.3–0.5	1	0.2–0.5
5-Roll	Hebels (1990)	Weiss (1977)	Potjer (1992)		
$V_{2,1}$	1.04	1	1.01–1.08		
$V_{3,1}$	0.26	0.7–0.9	0.8–0.95		
$V_{4,1}$	0.065	0.2–0.5	0.04–0.12		
$V_{5,1}$	0.03	0.2–0.5	0.01–0.03		

In pond-fed coaters, the coated film thickness is directly proportional to the metering gap, g_n, and to the metering gap flow rate factor, λ_n; moreover, the coated film thickness remains constant if the roll speed ratios remain constant. The coated film thickness depends on all $n+1$ roll speeds (n roll speed ratios). This is a large parameter space to explore. So, sets of roll speeds that reflect common practices were used as test cases. Table 1 shows the results of a survey of industrial roll coater practitioners and available literature.

Apart from the consensus that the roll speeds generally increase from slower moving gate rolls to the backing roll (roll 1), there is no general agreement on the distribution of intermediate roll speeds. Roll 2 is commonly run at or near line speed because application takes place at gap_1.

Before turning to these test cases, it is instructive to examine some generic cases of roll speed ratios. These illustrate how thin films are created in a multiple roll transfer coater. Two mechanisms work to create thin films: multiple film-splitting, and the speedup of the liquid films on successive rolls.

Effect of Film-Splitting. The number of rolls determines the number of film-splits and thus, to a variable extent, the thinness of the coating. The coated film thickness, in units of $\lambda_n g_n$, becomes a function solely of the number of rolls, $n+1$, when all of the roll speeds are equal ($V_{i,1}=1$; $\alpha=1$; $i=2, n+1$):

Ordinary Transfer Roll Coater

$$\frac{h_1}{\lambda_n g_n} = \frac{1}{2n}, \tag{20}$$

Doctored Transfer Roll Coater

$$\frac{h_1}{\lambda_n g_n} = \frac{1}{2^n}. \tag{21}$$

Figure 5 shows the relationship between the dimensionless coated film thickness $h_1/\lambda_n g_n$ and the number of rolls. The first few additional rolls sharply reduce the coated film thickness in both the ordinary and the doctored types of coaters. But the two types differ significantly when more than three rolls are used. The doctored roll coater produces a much thinner coating than the ordinary transfer coater with the same number of rolls. In fact, a four-roll, doctored system coats as thin as a five-roll ordinary system (all roll speeds equal). The lower graph of Figure 5 (logarithmic scale) illustrates how adding rolls beyond the eighth or tenth of an ordinary system yields little thinning of the coated film, whereas adding rolls to a doctored roll system continues to yield substantial thinning. So if the goal is to create a thin coating, it is evident that doctoring the recycle films lowers the number of rolls needed. It also reduces greatly the residence times of portions of the liquid in the multiple-roll system.

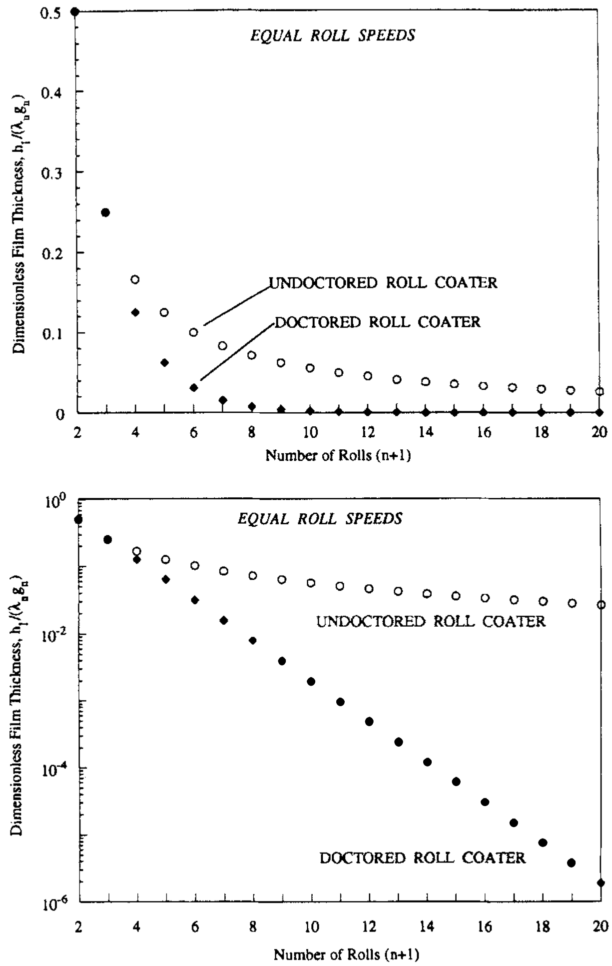


Figure 5. In pond-fed transfer coaters with equal roll speeds, the coated film thickness falls as the number of rolls is increased.

Adding rolls beyond the eighth or tenth roll of an ordinary system yields little thinning of the coated film, whereas adding rolls to a doctored-roll system continues to yield substantial thinning. A logarithmic scale is used in the lower graph.

Figure 6 shows the intermediate film thicknesses in an ordinary and a doctored five-roll system when all roll speed

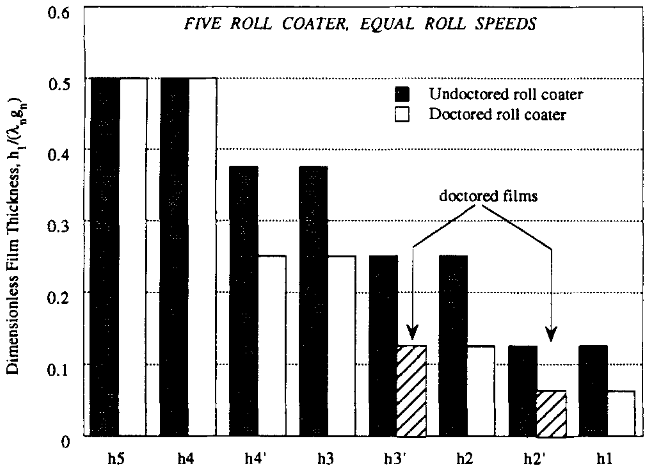


Figure 6. Intermediate film thicknesses fall from the feed nip to the application nip.

The two cases shown are doctored and undoctored five-roll coaters with equal roll speeds (h'_2 and h'_3 are the doctored films).

ratios are set to unity. With the doctored system, the film thickness is halved at each film-split, whereas with the ordinary system, the film thickness does fall but is not quite halved because of the recycled films feeding the intermediate gaps. The five-roll doctored coater requires two recycle streams to be removed: h'_2 and h'_3.

Effect of Roll Speedup. When the roll speeds are not equal, and instead increase from the slower moving gate rolls to the faster moving line speed, the liquid is accelerated at each transfer, and this causes the films to thin further. Figure 7a illustrates the effect: it shows the coated film thickness in units of $\lambda_n g_n$ as a function of the ratio of the end roll speed to the line speed, $V_{n+1,1}$, for $\alpha = 1$ and $\beta = 2/3$. Because no consensus on the sequence of the intermediate roll speeds was found, they were arranged in arithmetic and in geometric series from $V_{n+1,1}$ to 1.

Geometric Series of Roll Speeds:

$$V_{i,1} = V_{n+1,1}^{[(i-1)/n]} \qquad i = 1, n+1. \tag{22}$$

Arithmetic Series of Roll Speeds:

$$V_{i,1} = V_{n+1,1} + \frac{(1 - V_{n+1,1})(n+1-i)}{n} \qquad i = 1, n+1. \tag{23}$$

Some examples of such distributions are given in Table 2.

In the two-roll coater, the backing roll is also a gate roll; so increasing the relative speed of this roll (or decreasing the speed of the end roll) causes the coated film thickness to rise. Coaters with more than two rolls show the opposite behavior: the coated film thickness falls as the end roll speed decreases. At low gate roll speeds, $V_{n+1,1} \leq 0.2$, the geometric progression of intermediate roll speed ratios produces a much thinner coated film than the arithmetic progression does. But the limiting case $V_{n+1,1} = 0$ is not realistic in the geometric progression because it implies that all intermediate roll speeds are zero.

If there is a limit on the roll speed ratio between any two adjacent rolls, this sets the minimum possible end roll speed. For example, if for some reason the roll speed ratio limit were 4:1 (a value chosen by Hebels, 1990), the minimum $V_{n+1,1}$ in a two-roll coater would be 0.25; in a three-roll coater in arithmetic progression, 0.143; in a three-roll coater in geometric progression, 0.0625. The diamond-shaped symbols on the curves in Figure 7 denote the minimum end roll speed for a roll speed ratio limit of 4:1. Much slower end roll speeds, and corresponding thinner coat weights, are possible with a geometric progression of roll speeds. Moreover, the doctored roll coaters produce much thinner films for a given number of rolls at any end roll speed, regardless of the intermediate roll speeds.

Whereas Figure 7a is for the film-splitting exponent $\beta = 2/3$ a large value for this parameter, Figure 7b is for $\beta = 1/3$, which is near the low end of the range for metering gaps between rigid rolls (still lower values may arise when one of the rolls is rubber-covered (Carvalho and Scriven, 1993)). At the lower value of the film-splitting exponent the trends are all the same, but the coated film thickness is 10 to 15% lower.

Effect of Varying a Single Roll Speed. Equations 14–19 show that the final coated film thickness depends on every roll speed ratio in a system. Figure 8 and Figure 9 illustrate how the coated film thickness changes when one roll speed is changed from its base value, and all the others are kept the same. What is shown is the coated film thickness as a fraction of its base value vs. the roll speed as a fraction of its base value. The base roll speed ratios and the base coated film thickness (in units of $\lambda_n g_n$) are indicated in the insets of the graphs.

Figure 8 shows how the coated film thickness changes as individual roll speeds vary about the base case of equal roll speeds. The coated film thickness generally lies between 0.5 and 1.25 times the base value when the individual roll speeds are adjusted from 0.5 to 2 times their base value. Some rolls have a greater effect on the coat weight than others.

The top four plots pertain to undoctored coaters, which are most affected by the speeds of roll 2 and roll 1. Coaters with five or more rolls are affected the same by rolls 2 through $n-1$. The bottom two plots pertain to doctored coaters, which are most affected by roll n. The curves for the four-roll coater and the five-roll coater are identical. In fact, just four curves describe how a doctored coater depends on the individual roll speeds, regardless of the total number of rolls $n+1$. The four curves are for roll $n+1$, roll n, rolls 2 through $n-1$, and roll 1. These generalities apply only to the base cases of equal roll speeds, but similar trends are apparent in the industrial examples shown in Figure 9.

Certain roll speed settings in Table 1 are the base cases of the curves in Figure 9. The three-roll coaters coat much thicker and have much smaller latitude in attainable coat weights than the four-roll coaters, and likewise for the four-roll coaters compared to the five-roll coaters.

In almost every case the coated film thickness is most sensitive to V_n—the speed of roll n. In the five-roll coater (after Potjer, 1992), a nearly linear relationship between the coated film thickness and V_n is predicted (lower left graph of Figure 9); this linear relationship has been reported elsewhere also

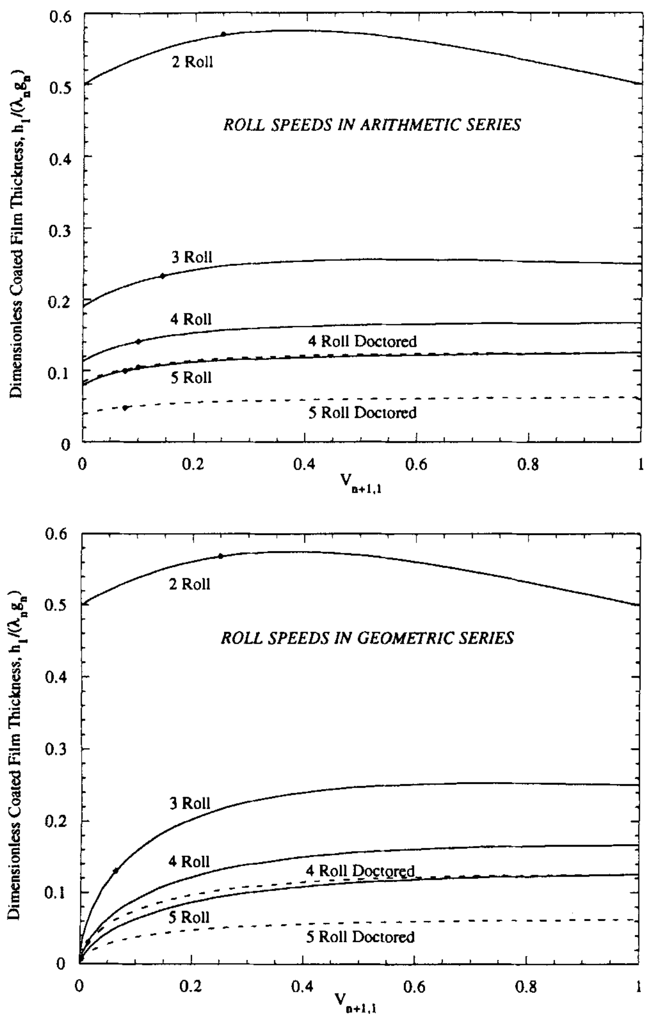


Figure 7a. As the ratio of the end-roll speed to the line speed $V_{n+1,1}$ decreases, the coated film thins due to speedup of the liquid.

The intermediate roll speeds are arranged in an arithmetic progression (top) and a geometric progression (bottom) from $V_{n+1,1}$ to line speed of unity. The symbols ♦ indicate the minimum $V_{n+1,1}$ allowed by a 4:1 limit on speed ratio between any two rolls; $\beta = 2/3$.

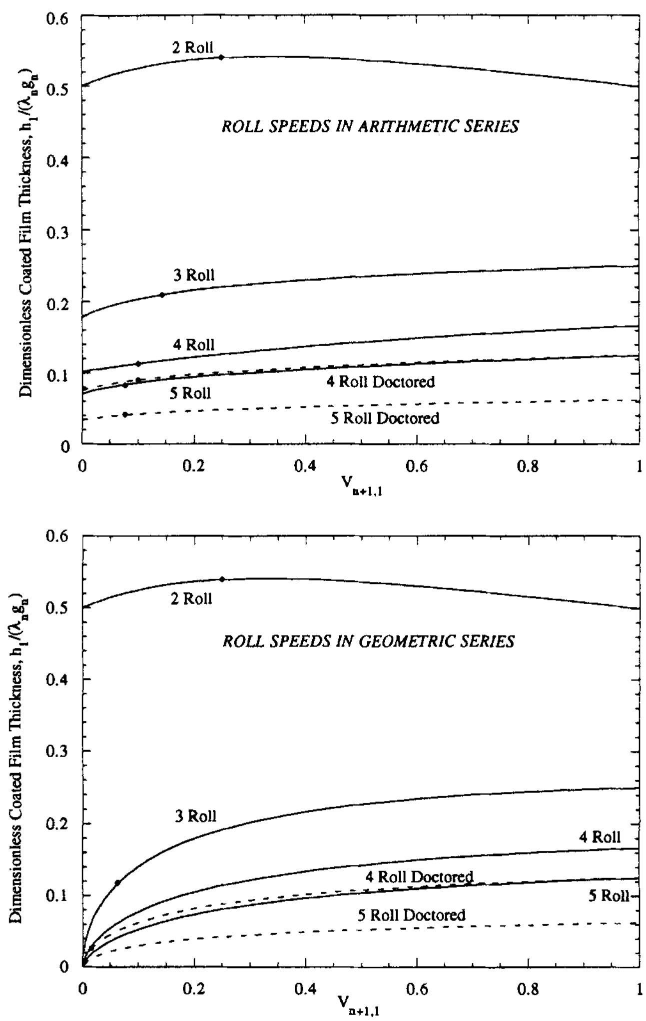


Figure 7b. Same as Figure 7a except $\beta = 1/3$.

The coated film is 10–15% thinner in all cases with the smaller film-splitting power.

(Hebels, 1990). The experimental data of DeSanti and Ostness (1992) from three- and five-roll coaters, although limited, is in general agreement with the theoretical predictions offered here.

Effect of Varying More Than One Roll Speed. Coated film thickness contour plots like those in Figures 10–17 illustrate the result of changing two roll speeds simultaneously. In Figure 10, the contours of coated film thickness in the plane of $V_{n+1,1}$, $V_{n,1}$ [the plane of $V_{5,1}$, $V_{4,1}$ in the case of the five-roll coater (top right) and the plane of $V_{4,1}$, $V_{3,1}$ in the case of the four-roll coater (lower left)] both show a global maximum in the coated film thickness. The maximum of the four-roll coater is near $h_1/g_n\lambda_n = 0.194$ at $V_{3,1} = 0.85$, $V_{4,1} = 0.3$. The maximum of the five-roll coater is near $h_1/g_n\lambda_n = 0.151$ at $V_{4,1} = 0.6$, $V_{5,1} = 0.2$. Figure 11 shows the same contour plots but with a low film-splitting exponent of $\beta = 0.3$. The shapes of the contours are similar, but the coated film thicknesses

Table 2. Examples of Geometric and Arithmetic Series of Roll Speeds of Multiple Roll Coaters

	5-Roll				4-Roll				3-Roll			
	Geometric		Arithmetic		Geometric		Arithmetic		Geometric		Arithmetic	
$V_{1,1}$	1	1	1	1	1	1	1	1	1	1	1	1
$V_{2,1}$	0.56	0.84	0.78	0.88	0.46	0.79	0.7	0.83	0.32	0.71	0.55	0.75
$V_{3,1}$	0.32	0.71	0.55	0.75	0.22	0.63	0.4	0.67	0.1	0.5	0.1	0.5
$V_{4,1}$	0.18	0.59	0.33	0.63	0.1	0.5	0.1	0.5				
$V_{5,1}$	0.1	0.5	0.1	0.5								

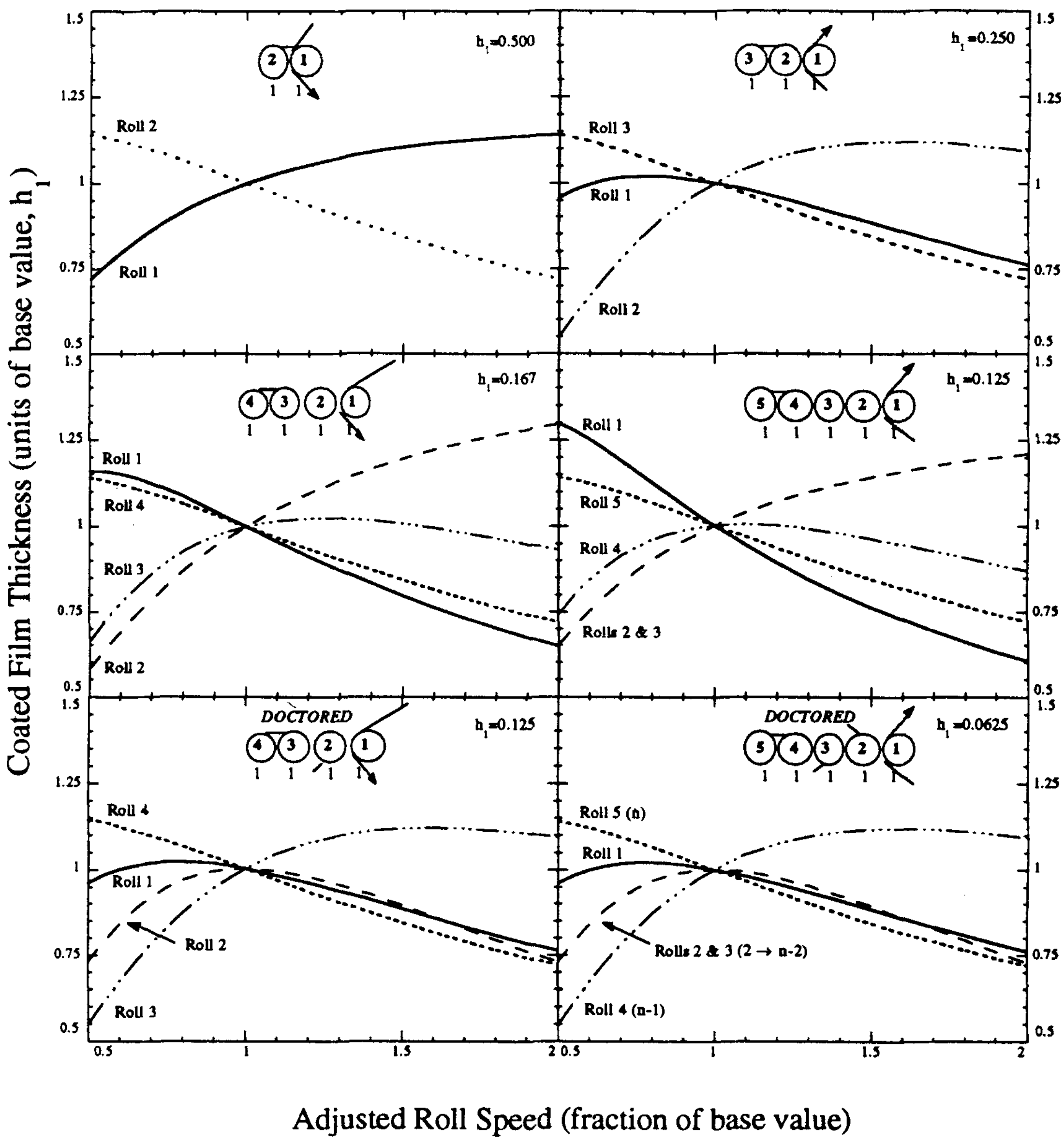


Figure 8. Coated film thickness changes as a single roll speed is adjusted while all other roll speeds are kept constant.

The adjusted roll speed and film thickness are given as a fraction of their base value. (Base values, all unity, are shown in the insets; h_1 is in units of $\lambda_n g_n$; $\alpha = 1$, $\beta = 2/3$.)

are 10–15% lower, and the global maximums are no longer present.

The contour plots in Figures 12 and 13 are all for a five-roll, pond-fed, ordinary (undoctored) coater. The six possible roll speed ratio combinations are displayed. In any given plot, two of the four speed ratios are changed and two remain constant at their base case values, namely $V_{2,1} = 1.05$, $V_{3,1} = 0.85$, $V_{4,1} = 0.1$, $V_{5,1} = 0.03$. The six contour plots show that in most cases $V_{4,1}$ has the greatest effect on the coated film thickness and that in every case $V_{2,1}$, $V_{3,1}$, and $V_{5,1}$ provide very little control over the coated film thickness.

Film thickness contours of a four-roll doctored coater are shown in Figures 14 and 15. The upper left graph is in the $V_{4,1}$, $V_{3,1}$ plane at constant $V_{2,1}$ ($V_{2,1} = 1$). The shape of the contours is the same as that of the contours of the three-roll coater (upper left graph of Figure 10) except for the scale of the contour levels. This is also made evident by comparing Eqs. 15 and 18 for coated film thickness. The effect of changing $V_{2,1}$ is the opposite in the doctored and undoctored cases. Whereas raising $V_{2,1}$ thickens the coated film in the undoctored case, it thins the coated film in the doctored case.

Figures 16 and 17 show film thickness contours in the six possible roll speed ratio planes of a doctored five-roll coater (compare the five-roll coater in Figures 12 and 13). The contours in the plane of $V_{5,1}$, $V_{4,1}$ have the same shape as in the case of the three-roll coater—indeed, as in every doctored roll coater regardless of the number of rolls. As in undoctored coaters, roll n has the strongest control over the coated film thickness, and the other rolls have very little control. Like the other examples, the film-splitting exponent changes the contour shapes little, yet reduces the coated film thickness by 10–15%.

Reverse Roll Coaters

Reverse roll coaters utilize from two to five rolls, all corotating so that adjacent roll surfaces move in opposite directions. The earliest and simplest form is the two-roll, pond-fed reverse coater (Bradner, 1931) described below. Other forms use more rolls to separate the feeding, metering, distribution, and application functions to individual gaps or nips. The most popular forms are the three- and four-roll, pan-fed reverse coaters (Booth, 1970; Münch, 1932). These effectively separate the four coating functions (Benjamin and Scriven, 1992). Here we set up the equations for n reverse rolls and analyze the simplest forms of reverse-roll coaters.

Theory

A system of n corotating rolls in a linear arrangement is shown in Figure 18. The general mass balance equation at a gap or nip in the reverse-roll case is the same as in the forward-roll case, namely, Eq. 3. Slightly rewritten, it becomes

$$V_{i+1}(h_{i+1} - h'_{i+1}) = V_i(h_i - h'_i) \qquad 2 \le i \le n-1. \quad (24)$$

In the reverse-roll case, there is no simple equation relating h_i to h'_i, in contrast to the case of the forward roll film split. Instead, the film thickness must be related to the net

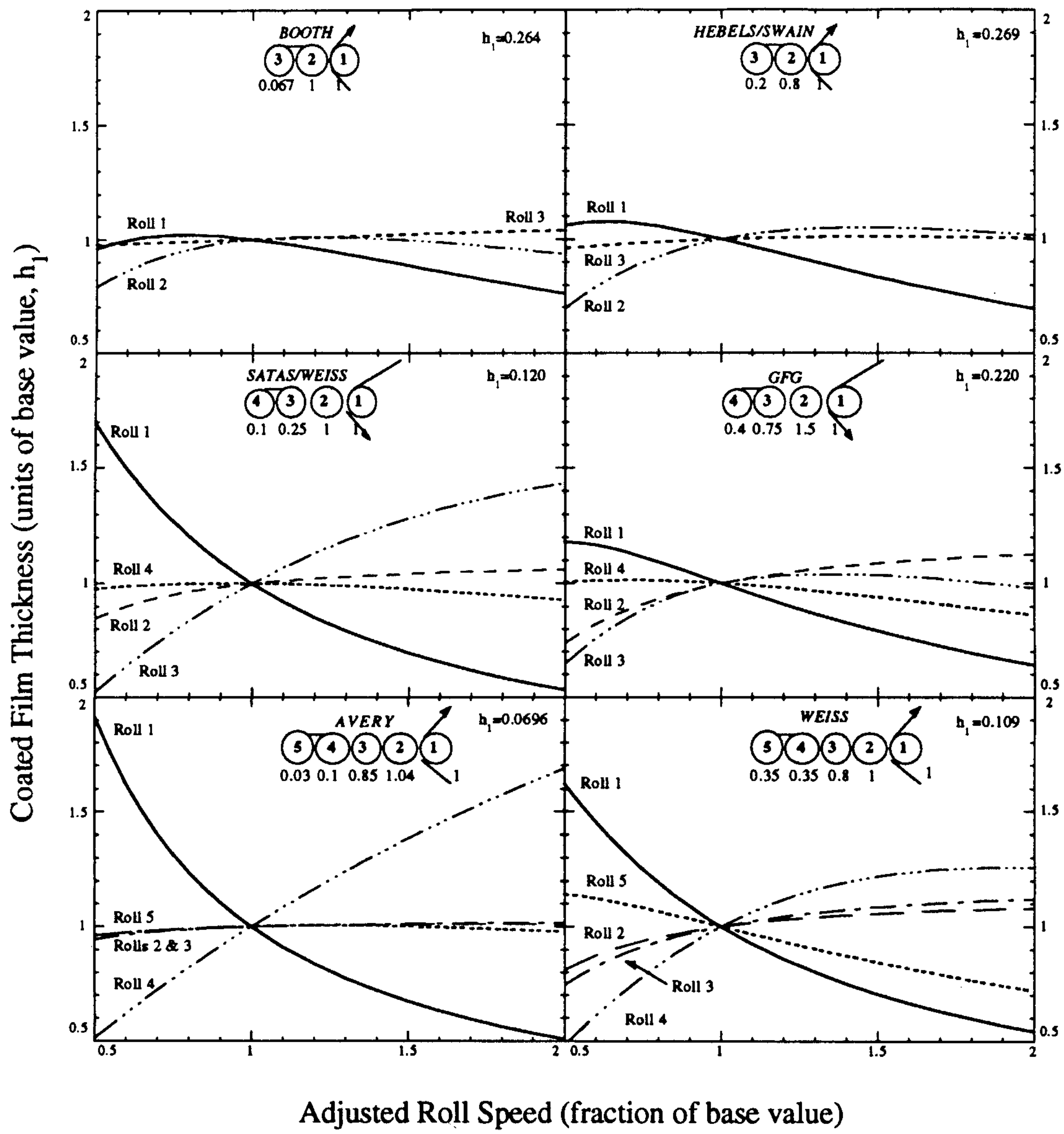


Figure 9. Coated film thickness changes when one roll speed is adjusted while all other roll speeds are kept constant for various three-, four- and five-roll coaters.

The adjusted roll speed and film thickness are given as a fraction of their base value. (Base values are shown in the insets; h_1 is in units of $\lambda_n g_n$; $\alpha = 1$, $\beta = 2/3$; GFG is Zahn's affiliation and AVERY is Potjer's.)

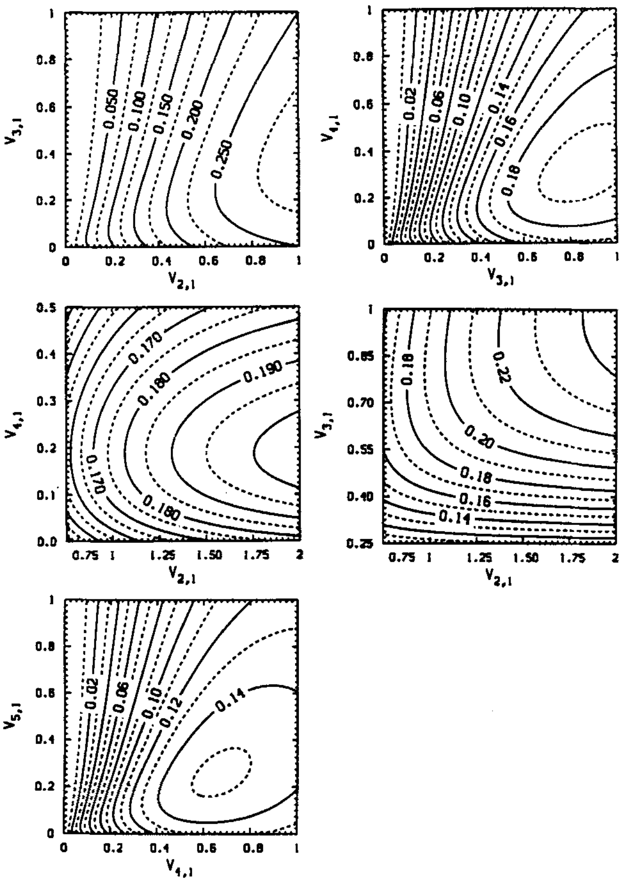


Figure 10. Coated film thickness contours (in units of $\lambda_n g_n$) of pond-fed, undoctored coaters (α = 1, $\beta = 2/3$).

Top/left: three-roll; top/right: four-roll ($V_{2,1}$ = 1); middle/left: four-roll ($V_{3,1}$ = 0.5); middle/right: four-roll ($V_{4,1}$ = 0.25); bottom/left: five-roll coater ($V_{2,1}$ = 1, $V_{3,1}$ = 1).

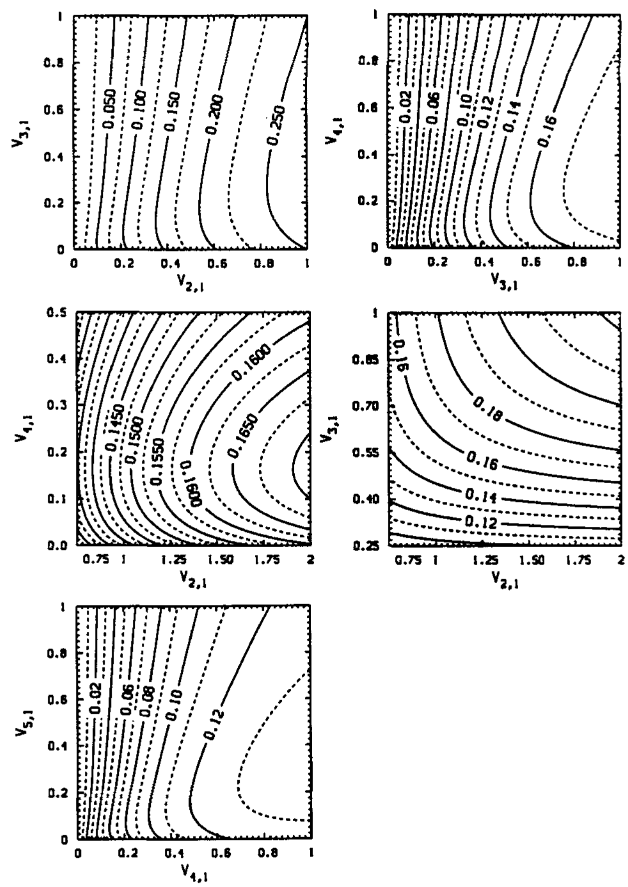


Figure 11. Same as Figure 10, except $\beta = 0.3$.

The trends are the same, but the shapes and values of the contours are slightly different.

flow rate through the gap, $Q^i_{\rm net}$:

$$V_{i+1}h_{i+1} - V_i h_i = Q^i_{\rm net} \qquad 2 \le i \le n-1. \tag{25}$$

$$V_{i+1}h'_{i+1} - V_i h'_i = Q^i_{\rm net} \qquad 2 \le i \le n-1. \tag{26}$$

At this writing, little information is available about the relationship between $Q^i_{\rm net}$ and the roll speeds, the gap, and the inlet film thicknesses of this type of double-film-transfer reverse-roll gap. However, if the gap were to be fully flooded, the flow rate would be related to the gap and the roll speeds by

$$Q^i_{\rm net} = \lambda_i g_i (V_i - V_{i+1}) \qquad 2 \le i \le n-1, \tag{27}$$

where λ_i is a linear function of the pressure difference across the gap. For a zero pressure difference, $\lambda_i = 4/3$.

At the application nip, the liquid is often transferred to the substrate by a reverse-roll transfer nip in which the substrate is impressed into a deformable backing roll. In this way, all but a thin lubricating layer of liquid is transferred to the web. If this lubricating film is negligibly thin ($Q^1_{\rm net} \approx 0$), the delivered film is either stretched or compressed, depending on the roll speed ratio, $V_{2,1}$. For this type of application nip, Eq. 25 reduces to

$$h_1 = h_2 V_{2,1}. \tag{28}$$

Equations 25, 27, and 28 provide $2(n-2)+1$ equations for $2(n-2)+1$ unknowns: $Q^i_{\rm net}$ ($i = 2, n-2$) and h_i ($i = 1, n-1$). A formula for the coated film thickness h_1 as a function of the number of rolls n, the feed film thickness h_n, and the parameters λ_i and g_i follows by induction:

$$h_1 = V_{n,1}h_n + \sum_{i=2}^{n-1} V_{i,1}\lambda_i g_i (1 - V_{i+1,i}). \tag{29}$$

Feed systems

The feed systems used with reverse roll coaters are the same as those discussed in connection with forward roll systems. For reverse roll coating the most popular ones are again pan feed and pond feed (Booth, 1970). The pan feed was discussed earlier; the reverse roll pond feed is described here.

Reverse Roll Pond Feed. The pond feed shown in Figure 4c requires a scraper or doctor blade to keep the liquid con-

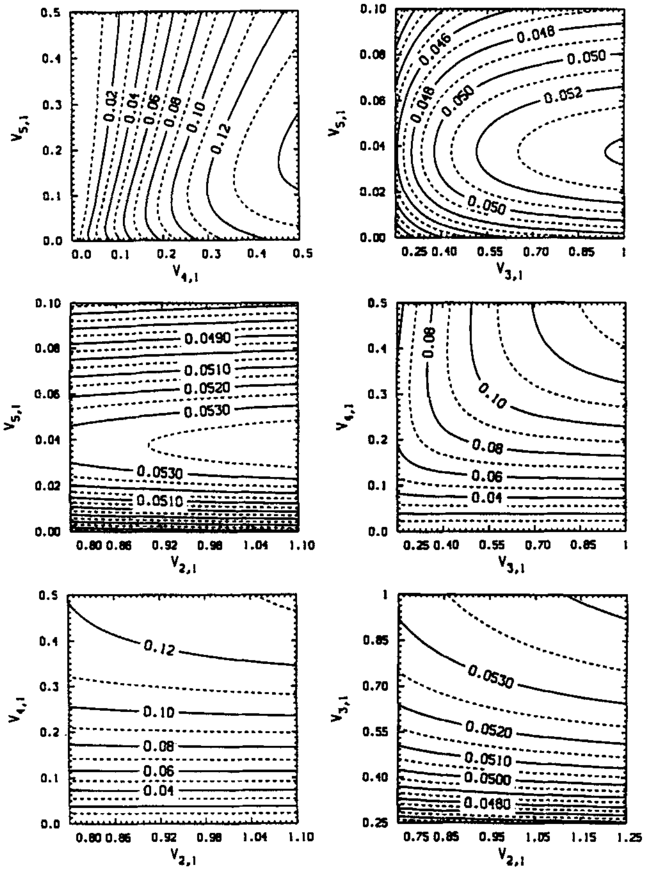


Figure 12. Coated film thickness contours (in units of $\lambda_n g_n$) of a five-roll, pond-fed, undoctored coater with base case roll speeds of $V_{2,1}=1.05$, $V_{3,1}=0.85$, $V_{4,1}=0.1$, $V_{5,1}=0.03$; ($\alpha=1$, $\beta=2/3$).

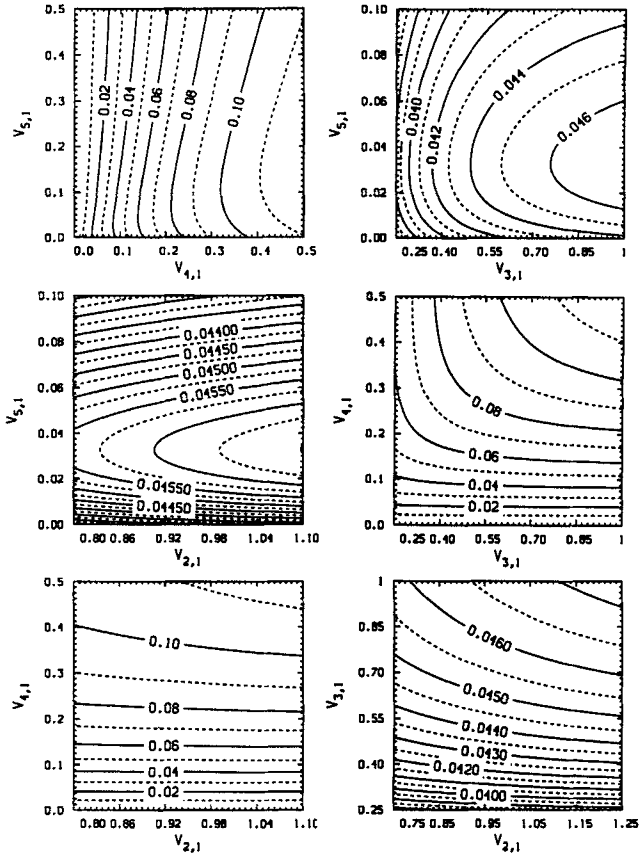


Figure 13. Same as Figure 12, except $\beta=0.3$.

The trends are the same, but the shapes and values of the contours are slightly different.

tained in the bank and to wipe the metering roll dry. As in the case of the forward roll pond feed, edge dams are required.

For the purpose of analysis, the reverse roll pond feed can be considered half-flooded. Coyle et al. (1990) and Benjamin (1994) found a relationship between the metered film thickness and the speed ratio, liquid properties, and roll gap for the half-flooded flow. This relationship is shown in Figure 19. To use it for the feed film thickness in a system model, it must be reduced to a simple equation form. The simplest form that adequately represents the theoretical results was found to be one that exponentially approaches a linear relation as the roll speed ratio approaches zero and exponentially approaches another linear relation as the roll speed ratio approaches infinity:

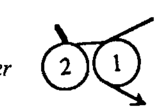
$$h_n = f_{\text{meter}}(V_{n+1,n}) = g_n \frac{e^{[c_1(c_2 - V_{n+1,n})]}(c_3 + c_4 V_{n+1,n}) + e^{[c_5(V_{n+1,n} - c_2)]}(c_6 + c_7 V_{n+1,n})}{e^{[c_1(c_2 - V_{n+1,n})]} + e^{[c_5(V_{n+1,n} - c_2)]}}. \quad (30)$$

This equation has seven parameters, c_1 through c_7, which are functions of the capillary number, $Ca \equiv V_n \mu/\sigma$. Those parameters are listed in Table 3 for $Ca = 0.01$, 0.1, and 1. The model predicts metered films that are thicker than the gap itself ($h_n/g_n > 1$), which signals the onset of an instability known as cascade or seashore (Bradner, 1931; Coyle et al., 1990).

Analysis of pond-fed reverse roll coaters

The two-roll, pond-fed configuration has only the one gap, which is described by Eq. 30:

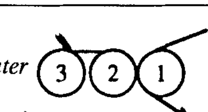
Two-Roll, Pond-Fed Reverse-Roll Coater



$$h_1 = g_1 f_{\text{meter}}(V_{2,1}). \quad (31)$$

Upon adding a third roll, the metering gap (gap_2) is separated from the application nip, and the coated film thickness becomes a function of the two-roll speed ratios, $V_{2,1}$ and $V_{3,1}$:

Three-Roll, Pond-Fed Reverse-Roll Coater



$$h_1 = g_2 V_{2,1} f_{\text{meter}}(V_{3,2}). \quad (32)$$

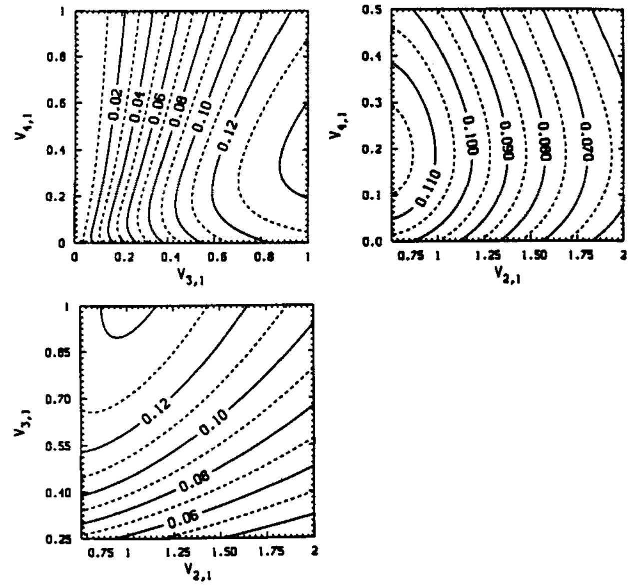


Figure 14. Coated film thickness contours (in units of $\lambda_n g_n$) of a four-roll, pond-fed, doctored coater with base case roll speeds of $V_{2,1}=1$, $V_{3,1}=0.5$, $V_{4,1}=0.25$; ($\alpha=1$, $\beta=2/3$).

To illustrate how the coated film thickness depends on the two-roll speed ratios, contours of h_1/g_n in the plane of $V_{2,1}$, $V_{3,2}$ were computed for $Ca=0.01$, $Ca=0.1$, and $Ca=1$ ($Ca \equiv V_2\,\mu/\sigma$). These contours are shown in Figure 20. The line drawn through the contours corresponds to the onset of the seashore instability as inferred from steady flow states; that is, $h_2/g_2 \geq 1$.

In all three cases, there are two metering roll speeds ($V_{3,2}$) for a given applicator roll speed ($V_{2,1}$) that give the same coated film thickness. The lower branch may be more stable because it is not as close to the seashore limit as the upper branch is. The range of possible coated film thicknesses h_1 (below the seashore limit) is about the same for $Ca=1$ and $Ca=0.1$: between 0.2 and 1.5 times the metering gap width g_2. At $Ca=0.01$, the range of h_1 is slightly smaller: 0.1 to 0.9 times the metering gap, and there is no seashore instability limit. To analyze pond-fed coaters with more than three rolls in a line requires more knowledge about the Q_{net} in a double-film-transfer flow than is currently available.

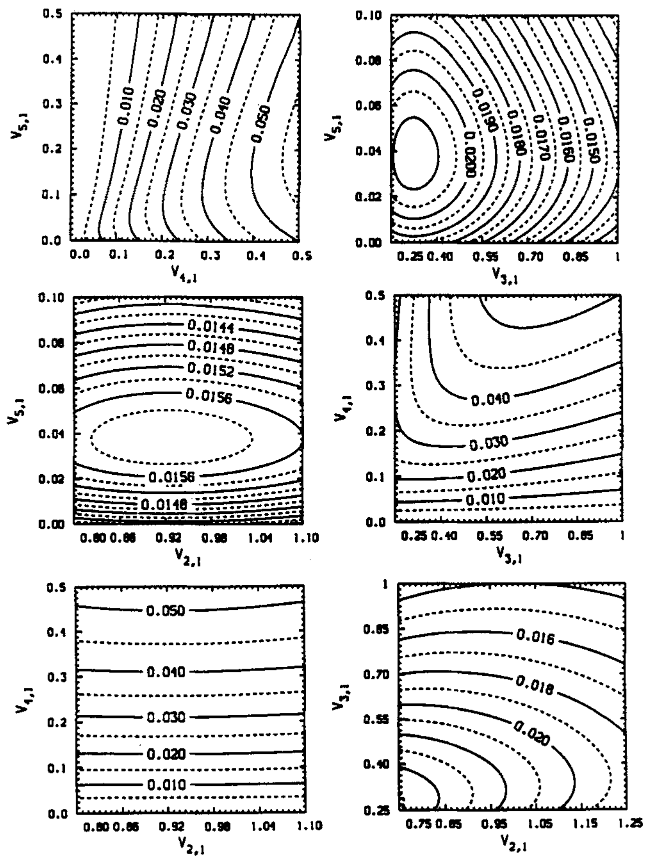


Figure 16. Coated film thickness contours (in units of $\lambda_n g_n$) of a five-roll, pond-fed, doctored coater with base case roll speeds of $V_{2,1}=1.05$, $V_{3,1}=0.85$, $V_{4,1}=0.1$; $V_{5,1}=0.03$; ($\alpha=1$, $\beta=2/3$).

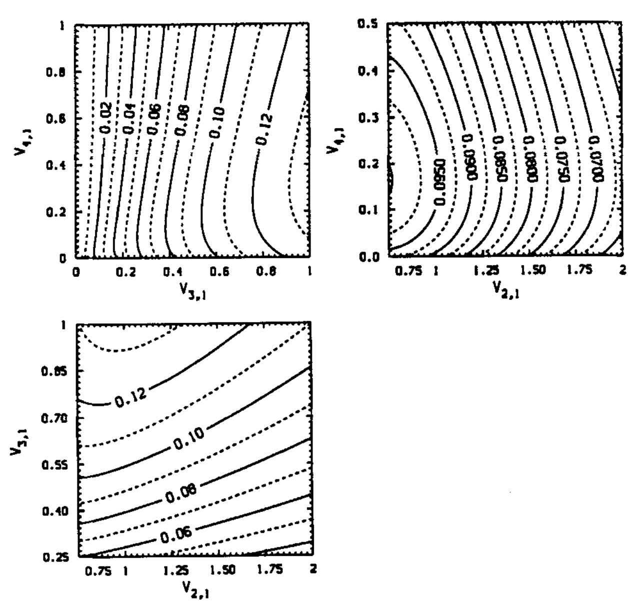


Figure 15. Same as Figure 14, except $\beta=0.3$.

The trends are the same, but the shapes and values of the contours are slightly different.

Analysis of pan-fed reverse roll coaters

Pan-fed reverse roll coaters differ from the pond-fed coaters in that the rolls are not usually in simple linear arrangements (Booth, 1970). In a three-roll, pan-fed reverse coater, roll 2 rotates in the pan and brings the feed film to the metering gap (gap_2) between rolls 2 and 3. If the feed film is sufficiently thick that the inlet to the metering gap is nearly flooded, the metering gap resembles the half-flooded metering gap of the pond feed. In this case the equation for the coated film thickness is the same as that for the three-roll, pond-fed configuration:

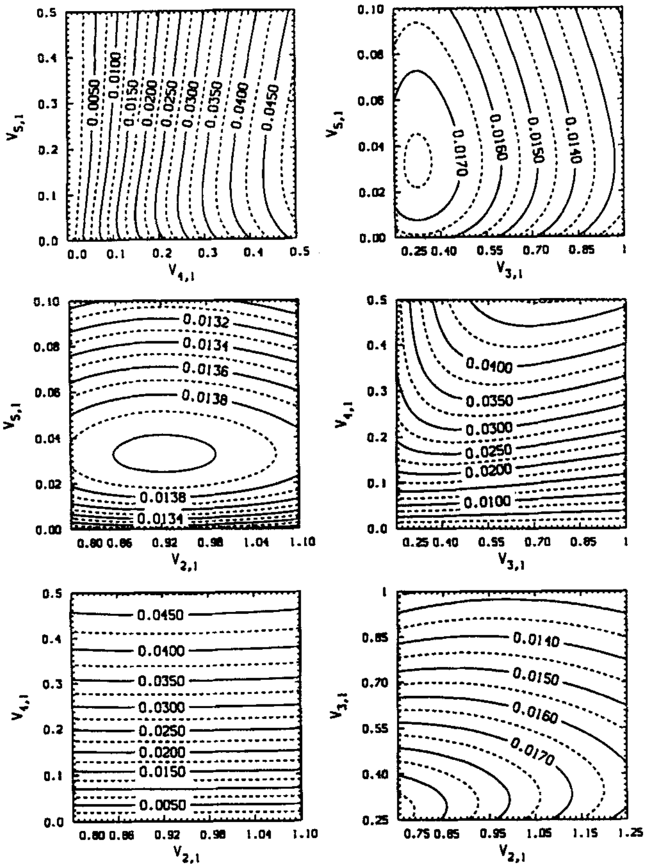
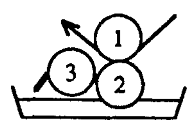


Figure 17. Same as Figure 16, except $\beta = 0.3$.

The trends are the same, but the shapes and values of the contours are slightly different.

Three-Roll, Pan-Fed Reverse Roll Coater



$$h_1 = g_2 V_{2,1} f_{\text{meter}}(V_{3,2}). \tag{32}$$

When the feed film is not sufficiently thick to flood the inlet of the metering gap, the metered film thickness becomes dependent on the feed film thickness as well. Indeed, the metering control actually moves further upstream and away from this gap. Unfortunately, there seems to be little published information about this type of film-fed metering gap, and in particular how the metered film thickness depends on the feed film thickness.

Table 3. Parameter Values of the Curve Fitted to Half-Flooded, Reverse Roll Metering Data

	$Ca = 0.01$	$Ca = 0.1$	$Ca = 1$
c_1	−0.0778	1.8620	8.9857
c_2	0.436	0.7550	0.4769
c_3	0.9569	0.6324	0.61169
c_4	−0.006734	−0.5591	−0.46615
c_5	2.8277	6.0829	12.01
c_6	−0.2908	−0.8797	0.02041
c_7	0.3018	2.5912	4.5583

Conclusion

Individual gaps in multiple roll systems—both forward and reverse—are coupled by conservation of coating liquid. If no liquid is lost from the ends of the gaps or rolls, the resulting mass balance equations are straightforward.

There are 11 basic types of gap flows between rolls. The flow rates through and the film-splits at the gaps, at least in the several important cases that have been analyzed by experiment and/or theory, are well approximated by simple formulas—power laws in cases of forward roll metering between rigid rolls, and hyperbolic-like functions in cases of reverse roll metering between rigid rolls. Similar formulas can be expected to describe the performance of gaps and nips between rubber-covered and rigid rolls. More information is needed about the relationships among flow rates, film thicknesses, roll speeds, gap widths, liquid properties, and roll cover properties, not only in the forward roll film-splitting flow and half-flooded reverse roll metering flow treated here, but also in double-film-transfer reverse roll flow, film-fed reverse roll metering flow, and double-film-fed forward roll flow.

In the configurations examined here, combining the mass balance equations with the approximate gap performance equations reveals that the coated film thickness is largely determined by the number of film-splits, thus the number of rolls used, and to a lesser extent by the roll speed ratios. In some cases a single roll speed can have multiple values that result in the same coated film thickness when all other speed ratios are left unchanged. In most cases, roll n, the roll next to the end, has the most control over the coated film thickness. In all cases doctoring each roll of the transfer coaters

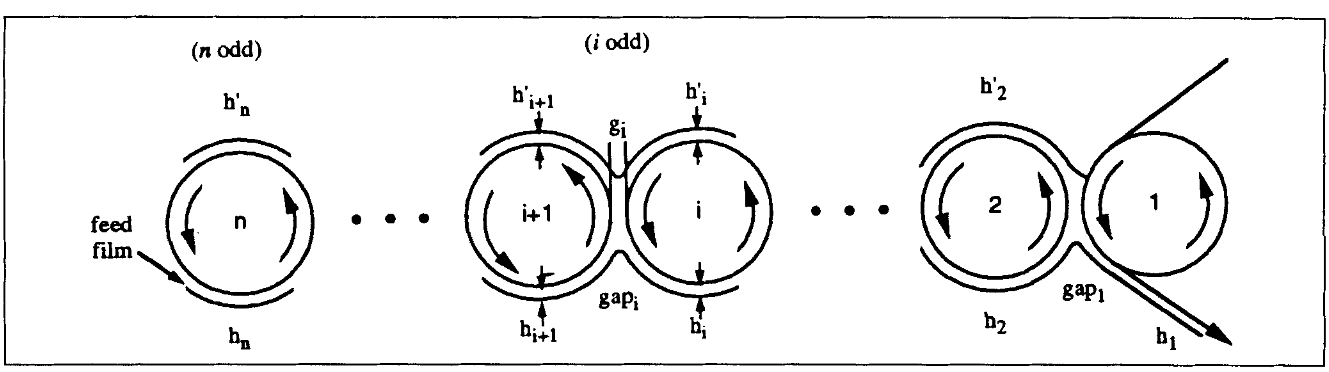


Figure 18. System of n reverse rolls in a linear arrangement with no doctored films.

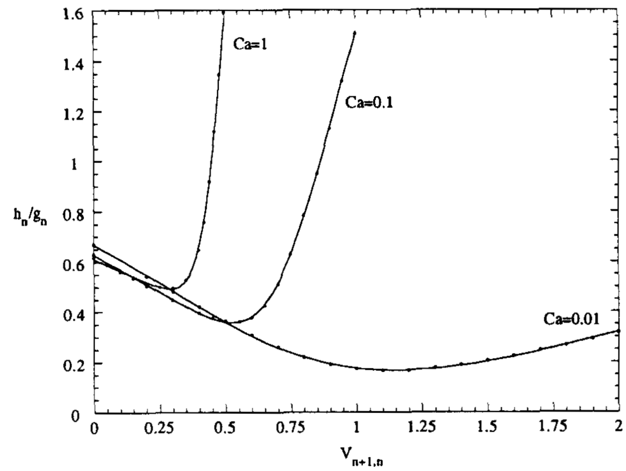


Figure 19. Feed film thickness (in units g_n) as a function of the ratio of speeds of roll n and roll $n+1$ at $Ca=0.01$, $Ca=0.1$, $Ca=1$.

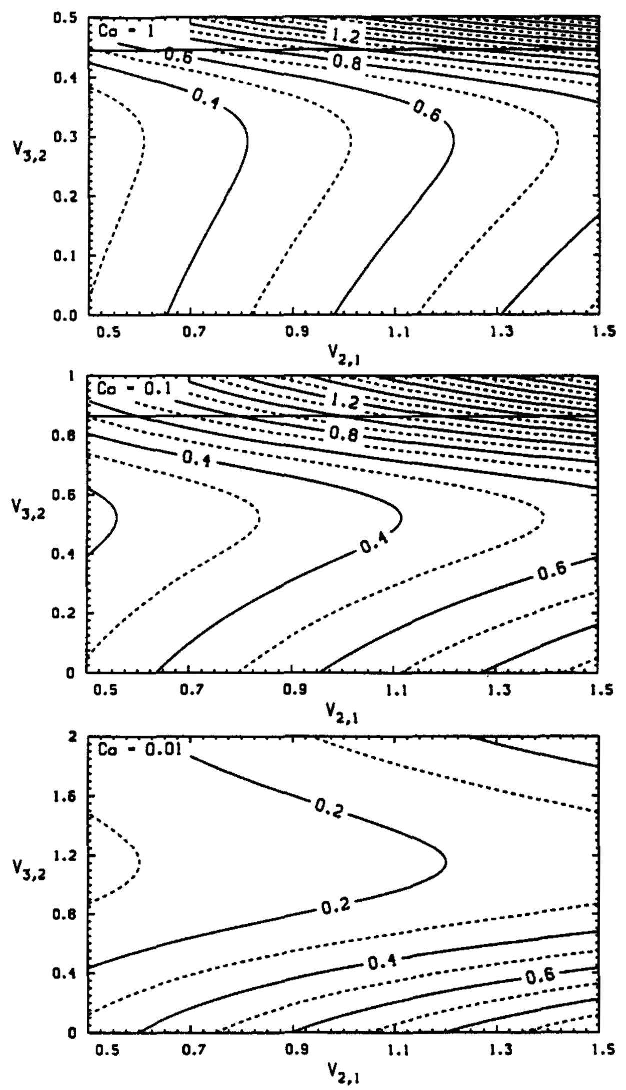


Figure 20. Contours of coated film thickness of a three-roll, pond-fed reverse coater in the plane of $V_{2,1}$, $V_{3,2}$.

produces a much thinner coating than the ordinary transfer coaters for the same number of rolls and the same roll speed settings. The film-splitting power exponent β, which is related to the relative magnitude and direction of the inertial and gravity forces, was found to have a small but significant effect. Changing β from 2/3 to 1/3 (low and high inertia limits, respectively) decreased the coated film thickness by 15% or less for all coaters examined.

The analysis of three-roll reverse coaters showed that two different applicator roll speeds can produce the same coated film thickness for a given line speed and metering roll speed. This is due to the nonmonotonic behavior of the metering gap flow. The capillary number based on the applicator roll speed is important in determining the range of possible coated film thicknesses.

The results establish how fundamentals can be brought to bear on understanding, comparing, predicting, and ultimately designing multiple roll systems. Many forms of roll coaters use various combinations of counter- and corotating rolls to combine both forward and reverse roll flows in one coater. Although we have not specifically addressed these types, it seems evident that they can be treated in the same way: mass balances combined with descriptions of the individual gap flows involved.

Acknowledgment

This research was funded in part by sponsors of the Coating Process Fundamentals Program of the Center for Interfacial Engineering at the University of Minnesota. The contribution of one of the authors (T.J.A.) was facilitated by a grant from the University's Undergraduate Research Opportunities Program. Thanks are due Bert R. Potjer of Avery Dennison and Robert M. Fridhandler of Polychrome for sharing their observations and experience with multiple-roll systems; their inputs helped focus this research.

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Manuscript received Aug. 27, 1993, and revision received June 13, 1994.